ON THE PROPAGATION OF WEAK WAVES IN A CONTINUOUS MEDIUM IN THE PRESENCE OF RADIANT ENERGY TRANSFER

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The problem of the non-adiabatic propagation of weak plane waves in a continuous medium was investigated in the papers of Prokof'ev [1,2], where a system of linearized equations of gas dynamics was written out, in which the effects of viscosity, heat conduction and radiation were taken into account and some solutions of those equations were derived. In this note, the propagation of weak plane waves is considered in the presence of radiant energy transfer only. Two particular cases are noted, namely, for the case of motion only slightly different from adiabatic or isothermal motions; and for these cases the amount of attenuation of weak waves was worked out.

The linearized system of equations describing the propagation of plane weak waves in the presence of radiation consists of the equation of conservation of mass, Euler's equation, equation of energy conservation, the equation of state, and the equation of radiant transfer (the latter equation is taken in the diffusion approximation):

$$\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial u}{\partial x} = 0, \qquad \frac{\partial u}{\partial t} + \frac{1}{\rho_0}\frac{\partial p'}{\partial x} = 0, \qquad \frac{\partial s'}{\partial t} + \frac{1}{\rho_0 T_0}\frac{\partial H}{\partial x} = 0$$
$$p = p(\rho, T), \qquad \frac{\partial^2 H}{\partial x^2} = 16\rho_0 \alpha \sigma T_0^3 \frac{\partial T'}{\partial x} + 3\alpha^2 \rho_0^2 H \tag{1}$$

where ρ', p', T', s', u and H are small incremental changes in the equilibrium values of density ρ_0, pressure p_0, temperature T_0, entropy s_0, velocity and radiant energy flux, α is the mass coefficient of absorption, and σ is the Stefan-Boltzmann constant.

Eliminating from the equations of the system (1) the velocity u, and replacing the variables ρ', s' by the variables T' and p' by use of the thermodynamic formulas

$$\rho' = \left(\frac{\partial \rho}{\partial T}\right)_p T' + \left(\frac{\partial \rho}{\partial p}\right)_T p', \qquad s' = \left(\frac{\partial s}{\partial T}\right)_p T' + \left(\frac{\partial s}{\partial p}\right)_T p'$$

we obtain the system

$$\begin{gathered}\left(\frac{\partial s}{\partial T}\right)_p \frac{\partial T'}{\partial t} + \left(\frac{\partial s}{\partial p}\right)_T \frac{\partial p'}{\partial t} + \frac{1}{\rho_0 T_0}\frac{\partial H}{\partial x} = 0\\ \left(\frac{\partial \rho}{\partial T}\right)_p \frac{\partial^2 T'}{\partial t^2} + \left(\frac{\partial \rho}{\partial p}\right)_T \frac{\partial^2 p'}{\partial t^2} - \frac{\partial^2 p'}{\partial x^2} = 0\\ 16 T_0^3 \rho_0 \alpha\sigma \frac{\partial T'}{\partial x} + 3\alpha^2\rho_0^2 H - \frac{\partial^2 H}{\partial x^2} = 0\end{gathered} \tag{2}$$

We shall seek for a solution of the system (2) including the dependence of the unknown functions T', p', and H on the variables x and t in the form $\exp[\,i(kx - \omega t)]$. From the condition of compatibility of the equations of the system (2) we obtain the dispersion equation, which after substitution in it of the expressions for the thermodynamic functions for a completely ionized ideal gas, takes the form

$$(16\alpha\sigma T_0^3 - i\omega c_p) k^4 + \left(i\frac{\omega^3 c_v}{a_T^2} - i3\alpha^2\rho_0^2\omega c_p - 16\alpha\sigma T_0^3 \frac{\omega}{a_T^2}\right) k^2 + i3\alpha^2\rho_0^2 c_v \frac{\omega^3}{a_T^2} = 0 \tag{3}$$

Here c_v and c_p are the specific heats of the completely ionized perfect gas at constant volume and constant pressure respectively, a_T and a_s are the isothermal and adiabatic velocity of sound respectively in the completely ionized gas. The solution of equation (3) gives the desired relation $k(\omega)$, and can be written out.

We notice two particular cases of the solution. If

$$\frac{16\alpha\sigma T_0^3}{c_p\left(\dfrac{\omega^2}{a_s^2} + 3\alpha^2\rho_0^2\right)}\frac{1}{a^2}\,\omega \ll 1$$

then

$$k = \frac{\omega}{a_s} + i\,\frac{16\alpha\sigma T_0^3}{c_p(\omega^2/a_s^2 + 3\alpha^2\rho_0^2)}\frac{\omega^2}{a_s}\left(\frac{1}{a_T^2} - \frac{1}{a_s^2}\right) \tag{4}$$

that is, the weak wave is an adiabatic sound wave with attenuation. If

$$\frac{c_p}{16\alpha\sigma T_0^3}\left(\frac{\omega^2}{a^2} + 3\alpha^2\rho_0^2\right)\frac{a_T^2}{\omega} \ll 1$$

then

$$k = \frac{\omega}{a_T} + i\left[\frac{c_p(\omega^2/a_T^2 + 3\alpha^2\rho_0^2)}{16\alpha\sigma T_0^3}\right]\frac{a_T}{2a_s^2}(a_s^2 - a_T^2) \tag{5}$$

that is, the weak wave is an isothermal sound wave with attenuation.

Comparing (4) and (5) with the solution of the problem of the dispersion of sound in the presence of thermal conductivity [3], it can be seen that in relation to the attenuation of sound, radiant energy is equivalent to thermal conductivity with a coefficient of thermal conductivity equal to

$$\chi = \frac{16\alpha\sigma T_0^3}{c_p(\omega^2 / a^2 + 3\alpha^2\rho_0^2)}$$

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